<u>Composition of Transformations:</u>

When two or more transformations are combined to form a new transformation, the result is called a *composition of transformations*. Since translations and reflections are both isometries, a glide reflection is also an isometry.

Just as we can have a composition of functions, so too can we have a composition of transformations. The first transformation produces an image, then the second transformation is performed on that image. The symbol for a composition of transformations is the same as for a composition of functions. For example, a rotation of 90° followed by a rotation of 180° would be indicated by $R_{180^\circ} \circ R_{90^\circ}$. Just as with functions, it is important to realize which transformation is performed first. $R_{180^\circ} \circ R_{90^\circ}$ is read as "a rotation of 180° following a rotation of 90°." In this particular case, you would get the same result regardless of which rotation was performed first. However, this is not always the case.

*** To remember which transformation to perform first, replace the composition symbol and what follows it inside parenthesis. Then work from the inside out. For example, when you are given $R_{180^\circ} \circ R_{90^\circ}$, replace the $\circ$ with parenthesis around R_{90° as shown:

$$R_{180^\circ} \circ R_{90^\circ} \quad \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow R_{180^\circ}(R_{90^\circ})$$

Now it is clear that the 90° rotation must be done first!

<u>Complete the following example:</u>

The coordinates of ΔFUN are F(-5,1), U(-1,1), and N(-1,7).

a) On a coordinate plane, draw and label ΔFUN.
b) Draw and label ΔF'U'N', the image of ΔFUN after $r_{x\text{-axis}}$.
c) Draw and label ΔF''U''N'', the image of ΔF'U'N' after $r_{y\text{-axis}}$.
d) What single transformation is equivalent to $r_{y\text{-axis}} \circ r_{x\text{-axis}}$?

<u>Glide Reflections:</u>

A *glide reflection* is a composition of a line reflection and a translation that is parallel to the line of reflection or vice versa. A glide reflection is an opposite isometry.

<u>For example:</u>

The coordinates of ΔGLD are G(1,-5), L(6,-4), and D(3,-1).

a) Draw and label ΔGLD.
b) Draw and label ΔG'L'D', the image of ΔGLD after $T_{-8,0}$.
c) Draw and label ΔG''L''D'', the image of ΔG'L'D' after $r_{x\text{-axis}}$.
d) What single transformation maps ΔGLD onto ΔG''L''D''?
e) Draw and label ΔG'''L'''D''', the image of ΔGLD after $T_{-8,0} \circ r_{x\text{-axis}}$. How does this image compare to ΔG''L''D''?

*** A glide reflection is a special composition since it is commutative.
*** Since a line reflection can be thought of as a “flip,” and a translation can be thought of as a “slide,”, then “a flip and a slide make a glide.”

Properties preserved (invariant) under a glide reflection:

1) Distance
2) Angle Measure
3) Parallelism
4) Colinearity
5) Midpoint

*** Orientation (lettering order) is not preserved under a glide reflection.